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Electrostatic quantum dots with designed shape of confinement potential

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Abstract

In electrostatic (gated) quantum dots, the potential confining the electrons is generated by the electrostatic field, which is created by the external voltages applied to the leads. Changing the geometry of the nanodevice we can obtain a diverse class of confinement potentials. We discuss the choice of the nanodevice parameters, which allows us to get the confinement potentials with the designed shape: from the rectangular potential well to the potential well with smooth edges. In particular, we find the conditions, under which the confinement potential possesses the Gaussian shape or is parabolic in a large region of the quantum dot.

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Keywords: Quantum dot; Confinement potential

1. Introduction

The profile of the potential confining the electrons in quantum dots (QDs) only in few cases is known with the sufficient precision [1–4]. Usually, the experimental data are interpreted with the use of the model confining potentials taken in forms of either the rectangular potential well or parabolic potential. The knowledge of the realistic profile of the confinement potential is necessary to a theoretical description of the electronic properties of QDs and—what is more important—to a fabrication of nanodevices with designed properties. In the present paper, we show a possibility of constructing the QD (i.e. the choice of geometric parameters) with the designed confinement potential profile.

2. Theory

We consider the nanodevice fabricated from the planar structure, which consists of undoped and doped GaAs and AlGaAs layers (Fig. 1(a)). The nanostructure considered well approximates the QDs studied by Ashoori [5–7].

The voltage applied between the gate and the substrate leads to the appearance of the electrostatic field in the nanodevice. The inhomogeneity of the field in the region below the cap with radius R generates the lateral confinement potential. The electrons are confined in the GaAs quantum well in the region below the cap (this is the physical QD region). We calculate the

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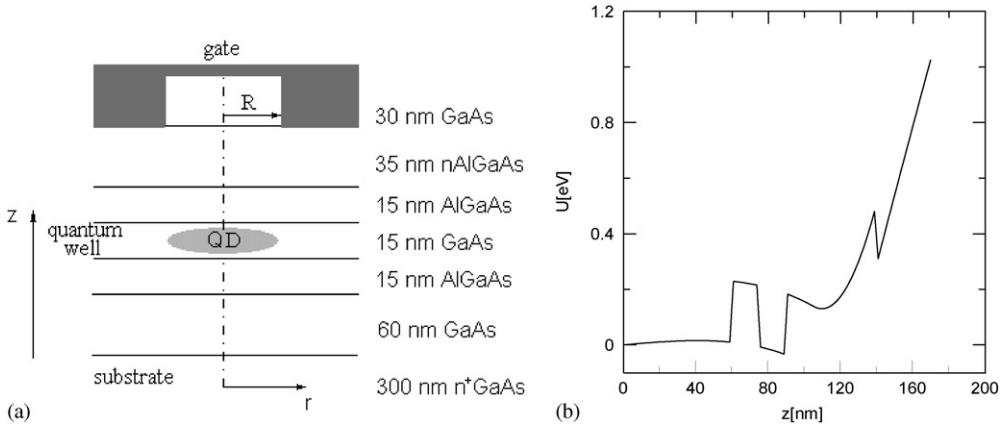

Fig. 1. (a) Schematic of the nanodevice. (b) Electron potential energy on the cylinder axis as a functions of z.

profile of the confinement potential $\varphi(r,z)$ by solving the Poisson equation in the cylindrical coordinates

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}\right)\varphi(r,z)=-\frac{\rho(z)}{\varepsilon_s\varepsilon_0}, \qquad (1)$$

where $\rho(z)$ is the charge density of the ionized impurities and ε_s is the static dielectric constant of GaAs. We put the boundary condition for the Poisson equation on the surface of cylinder, which encompasses the integration domain. The gate and substrate with the known potentials form the cylinder bases. We take on the cylinder radius large enough in order the electrostatic field to be approximately parallel to the cylinder axis on the side surface of the cylindrical integration domain. Then, we deal with the two-dimensional Poisson equation, which can be solved if we only know the potential on the leads. Poisson equation (1) has been solved by the finite-difference relaxation method.

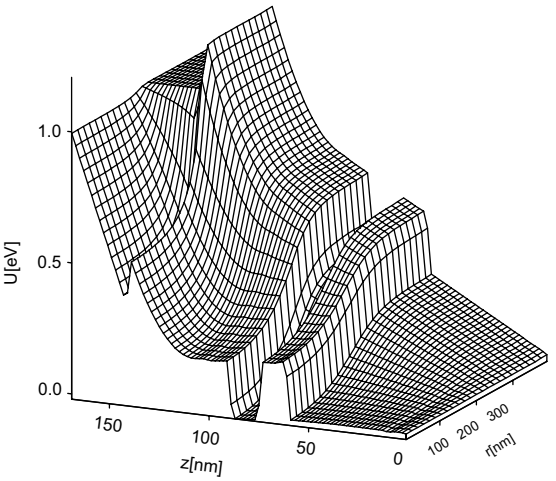

Fig. 2. Total potential energy of the conduction-band electron.

3. Results

Since both the electron charge and the voltage applied to the gate are negative, we present in figures—instead of the electrostatic potential—the potential energy of the electron, i.e. $U=-e\varphi$, where e is the elementary charge. The calculated profile of the electron potential energy is depicted in Figs. 1(b) and 2. Fig. 1(b) displays the potential energy on the cylinder axis and Fig. 2—the quasi-three-dimensional spatial profile of the potential energy. The results of Figs. 1(b) and 2 have been obtained for the same geometry of the nanodevice as in Ref. [5]. However, we have taken on the cap radius $R=205$ nm, which is greater than the nominal value (175 nm). Such choice of the cap radius allows us to reproduce the experimental results [5] of the capacitance spectroscopy in the magnetic field. Fig. 2 shows the results for the gate voltage $V_g=-0.375$ V, at which the binding of the first electron in the QD is observed [5]. If we move along the cylinder axis from the substrate to the gate, we observe [Fig. 1(b)] that the electron potential energy exhibits first the decrease and next the increase

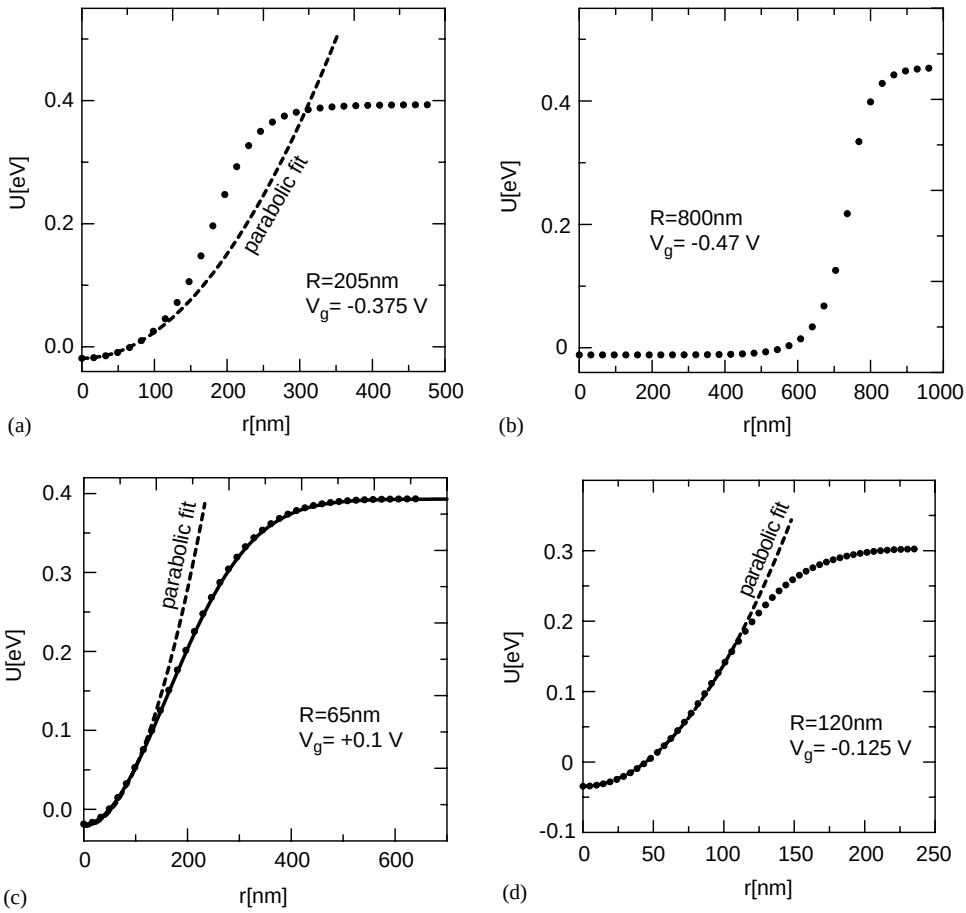

Fig. 3. Lateral confinement potential energy calculated from Poisson equation (dots), and fitted parabolic (dashed lines) and Gaussian functions (solid line) as a function of the distance from the cylinder axis for (a) the QD with parameters compatible with Ref. [5], (b) almost rectangular potential well, (c) Gaussian shape, and (d) almost parabolic in a large region of the QD.

modulated by the jumps, which result from the conduction band offsets. In the GaAs layer, the potential energy takes on negative values in the region near the cylinder axis. This is the QD region, in which the electrons are localized. In Fig. 3, we have displayed the electron potential energy $U(r, z_0)$ for z_0 taken on inside the quantum well region. These plots provide the profiles of the potential energy of the lateral confinement. The dots correspond to the numerical solutions of the Poisson equation and the solid and dashed curves are the plots of the adjusted analytic functions. The results of Fig. 3(a) have been obtained with the geometric parameters of the Ashoori [5] nanodevice. We see that—in this nanodevice—the lateral confinement potential is well approximated by the parabola in the small region near the cylindrical axis.

If we change the cap radius, we can obtain the different shapes of the confinement potential. For large radius we get the potential well with almost rectangular shape. Fig. 3(b) shows the potential energy for $R = 800$ nm, which correspond to the QDs studied in Ref. [6]. This shape of the confinement potential affects the capacitance spectroscopy results [6]. The flatness of the potential energy, which occurs in a large region near the QD center, causes that the electron energy is sensitive to the fluctuating potential, which is created by the ionized donors in the barrier layer. As a result, the electrons confined in the QD do not

form any well-defined shell structure. This leads to the bunching in the addition spectra [6].

It is interesting that we can obtain the confinement potential with a Gaussian shape. The results obtained for $R = 65$ nm are depicted in Fig. 3(c), which shows that the numerical results can be very well fitted by the Gaussian in the entire nanodevice. The properties of electrons confined in the Gaussian potential has been studied in Ref. [8]. Of course, near the QD center, the numerical results can be approximated by the parabola [cf. dashed line in Fig. 3(c)]. In Fig. 3(d), the confining potential obtained for $R = 120$ nm is depicted. For this cup radius the potential possesses an almost ideal parabolic shape in a large region of the QD.

The confinement potential energy shown in Fig. 3 has been calculated under the condition that exactly one electron is bound in the QD. The corresponding gate voltage values are $V_g = -0.375, -0.470, +0.100$ and -0.125 V for Figs. 3(a), (b), (c), and (d), respectively. All the other parameters of the nanodevice are the same.

In general, the dependence of the confinement potential on the parameters of the nanodevice is complex. When studying these dependences we have to repeat the calculations for each nanostructure. Nevertheless, there exist a certain important property of the confinement potential, which does not require the numerical calculations in order to be found. This is the scaling of the sizes and potentials of QDs. Looking at Eq. (1), we see that after multiplying all the geometric sizes of the nanodevice by factor k and simultaneously dividing the charge density by k^2 we obtain the solution of Eq. (1), i.e. electrostatic potential, as the same function of the coordinates, which are expressed in the length unit multiplied by k. Therefore, we know—without performing the calculations—that in the parabolicity region we obtain the k times smaller oscillator energy if we construct the nanodevice with k-fold enlarged linear sizes and k^2 times weaker doping of the barrier layer.

4. Summary

When fabricating the nanodevice from the fixed layer structure, the cap of certain size is put on the top layer. In such a manner, we fix the profile of the confinement potential, which can generate the QD region. In the present paper, we have shown the evolution of the confinement potential shape as a function of the cap radius from the nearly rectangular potential well to the potential well with smooth boundaries. In particular, we have determined the parameters of the QD, for which the confinement potential has a Gaussian shape and an almost ideal parabolic shape in a large region of the QD.

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