SCHEDAE INFORMATICAE
VOLUME 14 2005

Quantum Computing with Quantum Dots

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Abstract. A possible implementation of quantum computation on quantum dots is discussed. We focus on the application of the spin of the electron confined in the quantum dot to read/write operations of quantum bits (qubits). The realization of quantum logic gates is studied for the spin states of the two-electron system confined in the two coupled quantum dots. We also discuss the advantages and limitations of the quantum-dot quantum computing technology.

Keywords: quantum computation, quantum dot, qubit, quantum logic gate.

1. Introduction

In the last years we observe a rapid progress in the theory of quantum computing [1, 2]. Moreover, various physical realizations of quantum computations are intensively studied. The theory of quantum computation is based on a direct application of the laws of quantum mechanics to the computations. According to our present knowledge, quantum mechanics and quantum electrodynamics provide a complete description of the structure and properties of the world at the microscopic scale, i.e., the objects of sizes smaller or comparable with sizes of molecules. In particular, the quantum theory of atoms and elementary particles (electron, photon, etc.) describes with a high precision the properties, which are verified in experiments.

Quantum computational algorithms consist of sequences of logic operations, which are performed on quantum states and implement the quantized (discrete) values of basic physical quantities, like energy and angular momentum. The quantum information is recorded with the help of quantum bits, called qubits, which are the quantum states in the two-dimensional Hilbert space. The physical realization of the qubit can be done with the use of any two-level quantum system, i.e., the microobject characterized by the observable with two discrete eigenvalues. For example, the qubit can be realized on the two spin states of the electron or the two states of the polarization of the photon. The qubits serve to storage quantum information. They can be transformed with the use of quantum logic operations. In the mathematical language, the quantum logic operations (gates) are described by the unitary transformations between the quantum states.

Parallel to the progress in the theory of quantum computation the experimental studies are recently carried out in order to find physical realizations of qubits and to perform controlled transformations on them. Several different physical systems are investigated in order to develop the realizable quantum computing technology. The interesting results have been obtained for single ions in ion traps [3], atoms and photons in quantum-electrodynamics (QED) cavities [4], molecular systems in nuclear magnetic resonance (NMR) apparatuses [5], and Cooper pairs in superconductors [6]. Particularly promising is the application of semiconductor nanostructures, especially quantum dots, as the quantum computing devices. The semiconductor devices possess the advantage that the technology of their fabrication (nanotechnology) is a natural extension of the technologies used in the present computer industry and moreover they can be easily integrated with the existing hardware.

The quantum dots are the laboratory produced solid-state structures with nanometer sizes, in which the motion of charge carriers (electrons and holes) is limited in all three spatial dimensions. These are the smallest structures among the artificially fabricated objects. Their electronic properties can be modified and controlled by the modern electronic devices. We note that the quantum dots determine the limit of the current trend of miniaturization of electronic devices. This trend relies on the man-made producing of the devices with decreasing size. The smaller systems than quantum dots that can be used in future electronics (molecular electronics) are natural atoms and molecules. The quantum dots are called artificial atoms, since the confined electrons (holes) form localized quantum states with the properties similar to those of natural atoms. In particular, the energy levels associated with the quantum confined states are discrete. Applying an external electromagnetic field, we can change – in a broad range – the electronic properties of quantum dots. Therefore, the quantum dots are the nanostructures, which can be used as elements of future quantum computers.

The quantum computers are nowadays at the stage of the laboratory research. A different situation takes place for nanocomputers that are nowadays introduced into the production. The basic elements of nanocomputers, i.e., a nano-field-effect-transistor (NFET) and nano-integrated circuit (nanoIC), recently reach the size below 100 nm. Therefore, quantum phenomena appear in their operation. On the contrary to quantum computers, whose operation just exploits quantum effects, the quantum effects in nanocomputers play a damaging role and limit their computational efficiency. For example, the tunnel currents spoil the isolating properties of blocking layers. The operation of nanocomputers is still based on the laws of classical physics. We also note that some possible physical realizations of quantum computers, e.g., ion traps, QED cavities, and NMR systems, are of centimeter size, which suggests that these technologies will not necessarily lead to a further miniaturization of the future computing machines. However, it is expected that any technology of quantum computation should lead to an enormous increase of the computational power.

The history of the quantum computation begins with the articles of Feynman [7, 8], who as the first proposed a direct application of the laws of quantum mechanics to a realization of computational algorithms. We underly that the operation of up-to-date built computers, and also nanocomputers, can be completely described by the laws of classical physics, in particular the Maxwell equations, which are the fundamental equations of the classical electrodynamics. In spite of the fact that the present computers contain transistors, the operation of which is based on the electron band structure properties of semiconductors, the computations in the conventional computers run according to the equations of classical physics. For example, in conventional transistors the read/write operation of the single classical bit requires a flow of $10^6 - 10^9$ electrons. For comparison, the read/write operation of the quantum bit can be performed with the single electron.

The fundamental ideas of quantum computing were introduced and developed in the papers [9, 10, 11, 12, 13, 14, 15]. A model for quantum computations and a description of the universal quantum computer as a quantum Turing machine were elaborated by Deutsch [9]. Shor [10] introduced the quantum algorithm for the integer-number factorization. Grover [11] proposed the fast quantum search algorithm. Wooters and Zurek proved the non-cloning theorem, which puts definite limits on the quantum computations. Calderbank and Shor [14] elaborated the quantum error-correcting method. Recently, the theory of quantum computing is an advanced theory, which links the elements of physics, mathematics, and computer science [16].

In the present article, we discuss a possibility of application of quantum dots to quantum computations. The article is organized as follows: in Section 2 we provide a brief introduction to the quantum computing, in Section 3 we present basic properties of quantum dots, in Section 4 we discuss a possible realization of qubits and quantum logic gates on two-electron spin states in coupled quantum dots, and in Section 5 we give conclusions and summary.

2. Quantum bits and quantum logic gates

2.1. Qubits

The classical information is stored with bits, i.e., the states of the classical system, which take on two values 0 or 1, each of which occurs with probability 0 or 1. Quantum bits (qubits) are the quantum-mechanical counterparts of classical bits. The qubit is defined as a quantum state vector in the two-dimensional Hilbert space $\mathcal{H}^2$. If vectors $|0\rangle$ and $|1\rangle$ form the orthonormal complete basis in $\mathcal{H}^2$, then the qubit can be written down as

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle, \tag{1}$$

where the complex probability amplitudes c_0 and c_1 satisfy the normalization condition $|c_0|^2+|c_1|^2 = 1$. The set of states $\{|0\rangle, |1\rangle\}$ is called a computational basis.

The information capacity of classical and quantum bits is different. Contrary to the classical bit, which can be in classical state 0 or 1 with probability 1, the quantum bit takes on a continuum of values, which are determined by the amplitudes c_0 and c_1. However, these amplitudes are non-measurable. If we perform a measurement on qubit (1), we obtain either outcome 0 with probability $|c_0|^2$ or outcome 1 with probability $|c_1|^2$. However, if the quantum system is described by the qubit being exactly equal to one of the states of the computational basis, i.e., $|\psi\rangle = |0\rangle$ or $|\psi\rangle = |1\rangle$, then we can predict the exact result of the measurement with probability 1. This dichotomy between the non-observable general state of the qubit and the precise result of the measurement in the basis state (eigenstate of the observable) plays an essential role in quantum computations.

For the quantum computing, besides the single qubit states (1), we also need two-qubit states, which are the states of the two-particle quantum system. The two-qubit states can be constructed as tensor products of basis states $\{|0\rangle, |1\rangle\}$. Accordingly, the two-qubit basis consists of the states

$|00\rangle, |01\rangle, |10\rangle, |11\rangle$, where we apply a shortened notation, e.g., $|00\rangle \equiv |0\rangle|0\rangle \equiv |0\rangle \otimes |0\rangle$. The arbitrary two-qubit state has the form

$$|\Psi\rangle = c_0|00\rangle + c_1|01\rangle + c_2|10\rangle + c_3|11\rangle \ , \tag{2}$$

where the normalization condition takes on the form

$$|c_0|^2 + |c_1|^2 + |c_2|^2 + |c_3|^2 = 1.$$

2.2. Spin qubits

A particle with non-zero spin is particularly suitable for the physical realization of the qubit. The qubits can be formed from the spin states of the single electron, single nucleus, pair of electrons, or electron-hole system (exciton). In the present paper, we consider the particle with spin quantum number 1/2, e.g., the electron, for which the z component of the spin takes on the two values $\pm(\hbar/2)$. The operator of the z spin component is

$$s_z = \frac{\hbar}{2}\sigma_z \ , \tag{3}$$

where σ_z is the z Pauli matrix

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ . \tag{4}$$

The corresponding eigenequations have the forms

$$s_z|0\rangle = +\frac{\hbar}{2}|0\rangle \ , \ \ s_z|1\rangle = -\frac{\hbar}{2}|1\rangle \ . \tag{5}$$

The eigenstates can be written in the form of spinors, i.e.,

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \ |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ . \tag{6}$$

Another physical quantity of interest is the spin magnetic dipol, which possesses the z component

$$\mu_z = -\frac{1}{2}g^\star \mu_B \sigma_z \ , \tag{7}$$

where μ_B is the Bohr magneton ($\mu_B = 0.927 \times 10^{-23}$ Am^2), $g^\star$ is the effective Lande factor, which in semiconducting materials can take on positive as well as negative values, e.g., for the electron in Si $g^\star = 1.998$, in Ge $g^\star = 1.563$, and in GaAs $g^\star = -0.44$. For comparison, for the electron in the vacuum $g^\star = 2.0$.

The spin can be experimentally detected using the interaction of the spin magnetic dipol with the external magnetic field $\mathbf{B}$. For $\mathbf{B} = (0, 0, B)$ the Hamiltonian of this interaction has the form

$$H_{int} = -\mu_z B = \frac{1}{2} g^\star \mu_B \sigma_z B \; . \tag{8}$$

If the quantum system possesses energy E_ν in the absence of the external magnetic field, then – according to (3), (5), and (8) – the interaction of the spin magnetic dipol with the magnetic field leads to the splitting of this energy level into the two spin sublevels with energies

$$E_{\nu\pm} = E_\nu \pm \frac{1}{2} g^\star \mu_B B \; , \tag{9}$$

where sign + corresponds to state $|0\rangle$ with spin $+\hbar/2$ and sign $-$ corresponds to state $|1\rangle$ with spin $-\hbar/2$. Eq. (9) describes the spin Zeeman effect, which can be observed by the spectroscopic methods. For example, for Si at $B = 10$ T the spin splitting energy is $\sim$0.6 meV, which corresponds to the radiation with the wave length $\sim$2 mm.

2.3. Quantum logic gates

The qubits can be transformed using the quantum logic gates, which are performed with the help of unitary transformations U, which transform initial state $|\psi_i\rangle$ into final state $|\psi_f\rangle$ according to

$$|\psi_f\rangle = U|\psi_i\rangle \; . \tag{10}$$

Depending on the type of qubit, on which they operate, we deal with either one- or two-qubit gates. The quantum NOT gate, being a counterpart of the classical NOT gate, defined as

$$U_{NOT} \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; , \tag{11}$$

is an example of the one-qubit gate. If we write the one-qubit state (1) in a matrix form as

$$|\psi\rangle = \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} \; , \tag{12}$$

then the NOT gate operates on the one-qubit state as follows:

$$U_{NOT}\begin{pmatrix} c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_0 \end{pmatrix} . \tag{13}$$

As a result, the basis states $\{|0\rangle, |1\rangle\}$ have been interchanged, i.e., $|0\rangle \leftrightarrow |1\rangle$.

The two-qubit gate operates on the two-qubit state $|\beta_1, \beta_2\rangle \equiv |\beta_1\rangle|\beta_2\rangle$, where $\beta_1, \beta_2 = 0, 1$. An example of the important two-qubit gate is the controlled-NOT gate U_{CNOT}, for which the first qubit ($|\beta_1\rangle$) is the control qubit and the second qubit ($|\beta_2\rangle$) is the target qubit. The controlled-NOT gate transforms the two-qubit basis states as follows:

$$\begin{aligned} &U_{CNOT}|00\rangle = |00\rangle \ , \quad U_{CNOT}|01\rangle = |01\rangle \ , \\ &U_{CNOT}|10\rangle = |11\rangle \ , \quad U_{CNOT}|11\rangle = |10\rangle \ , \end{aligned} \tag{14}$$

which means that the CNOT gate changes the second qubit if and only if the first qubit is in state $|1\rangle$.

It was shown [13] that the set of logic operations, which consists of all the one-qubit gates and the single two-qubit gate U_{CNOT} is universal in the sense that all unitary transformations on N-qubit states, where N is arbitrary, can be expressed with the help of different compositions of the gates, which belong to the universal set of gates. Another important property of quantum computations is a quantum paralelism, which is based on the fact that the single unitary transformation can simultaneously operate on all the qubits in the system. The paralelism of quantum computations is an immanent characteristic of the quantum system; therefore, no special technology is necessary for its implementation.

2.4. Conditions for the physical realization of quantum computing

The quantum computing technology has to satisfy the following conditions: (i) the physical realizability of the qubits; (ii) the possibility of the precise preparation of the initial qubit state; (iii) the controlled unitary evolution of the qubit; (iv) the possibility of the accurate measurement of the final qubit state. When considering a physical system chosen to perform the quantum computations, we have to define the physical properties, which we will exploit in computations, e.g., spin of the electron, atomic nucleus, or photon, and the physical processes for the read/write operations, e.g., radiative transitions with the emission/absorption of photons.

The conditions listed above put certain limitations on the quantum computing technology. When designing the physical apparatus, which will perform the quantum computations, we face the problem of maintaining the controlled unitary evolution of the quantum system until the computations are completed. Such controlled evolution is possible provided that the quantum system is completely isolated from the environment. However, the complete isolation of the quantum-computing system disables the read/write operations. Therefore, some slight interaction of the quantum system with the environment is necessary. On the other hand, this interaction leads to decay and decoherence processes, which reduce the performance of the quantum computer.

In the decay process, the quantum system goes over – in a very short time – to a new state releasing a part of its energy to the environment. For example, the change of spin state $|0\rangle \to |1\rangle$ is accompanied by the emission of the photon. The decay is characterized by the decay time (relaxation time), which for the spin states can be very long. The recent measurements [17] of the Zeeman splitted spin states in quantum dots give a lower bound of 50 μs on the relaxation time at $B = 7.5$ T.

A decoherence is the much subtler effect, in which the energy is conserved but the relative phase of the different basis states of the qubit is changed. As a result of decoherence the qubit changes as follows:

$$|\psi\rangle \to c_0|0\rangle + e^{i\theta} c_1|1\rangle \ , \tag{15}$$

where the real number θ denotes the relative phase. The appearance of the non-zero relative phase results from the coupling of the quantum system with the environment and can lead to essential changes in the measurement statistics. For example, the quantum-mechanical expectation value of the measured quantity is changed. The decoherence time t_{decoh} is usually much shorter than the decay time; therefore, the decoherence can be treated as the most detrimental effect for the quantum computations. The ratio of the decoherence time t_{decoh} to the elementary operation time t_{oper}, i.e.,

$$R = \frac{t_{decoh}}{t_{oper}} \ , \tag{16}$$

is an approximate measure of the number of computation steps performed before the coupling with the environment destroys the qubit. For different quantum-computing technologies this ratio changes in broad limits [19]: $10^3 \leq R \leq 10^{13}$. For example, $R = 10^3$ for the electron states in quantum dots, $R = 10^7$ for nuclear spin states, and $R = 10^{13}$ for trapped ions.

3. Quantum dots

A semiconductor quantum dot [20] is the nanostructure, the linear size of which does not exceed 1 μm in each spatial direction. The typical sizes of the quantum dots are between $\sim$10 nm and $\sim$100 nm. The potential created in the quantum-dot nanodevice limits the charge carrier motion in all the three dimensions. This confinement potential possesses the range comparable with the size of the quantum dot and the finite depth. The typical depth of the confinement potential, i.e., the electron potential energy minimum measured with respect to the conduction band bottom of the embedding material, is of the order of $\sim$0.1 eV to $\sim$1 eV. This leads to the energy separations between the one-electron energy levels of the order of few meV. These energy separations put an additional limitation on the realizability of quantum computations, namely, in order to avoid thermal excitations, we have to maintain the temperature of the nanodevice below 1 K.

There are many types of quantum dots, among which, the best candidates for the possible implementation of quantum logic gates are the electrostatic (gate controlled) quantum dots. The electrostatic quantum dot [21, 22] consists of the sequence of vertically stacked layers, which form single or multiple potential wells and barriers. The source and drain electrodes are located at the bottom and top sides of the layer sequence. The entire quantum-dot nanodevice usually possesses a cylindrical symmetry and can have either a form of an etched pillar [23] or a layer sequence with a metal cap [24]. Depending on the number of barrier layers, the nanodevice can contain either a single or multiple quantum dot. In the pillar-shape quantum-dot nanodevice [23], an additional gate electrode is placed at the cylinder side, which increases the ability of tuning of the electrostatic field in the quantum dot. In the electrostatic quantum dot [21, 22], the confinement potential results from both the conduction band offsets and the external electrostatic field created by the electrodes. The knowledge of this potential is important for studying and modelling the electronic properties of the quantum dot. The confinement potential cannot be directly measured, but can be calculated from the first principles of electrostatics by solving the Poisson equation for the entire nanostructure. Such calculations were performed [21, 22] for the two different types of the electrostatic quantum dots. These were the pillar-shaped quantum dots of Tarucha et al. [23] and capped quantum dots of Ashoori et al. [24] The results obtained [22] show that the confinement potential V can be parametrized by either the Gaussian function [25] or power-exponential function [26] of the form

$$V = -V_0 \exp[-(r/R)^p - (|z|/Z)^p] \,, \tag{17}$$

where $V_0 > 0$ is the depth of the potential well, $r = \sqrt{x^2 + y^2}$, $p > 1$, R and Z are the measures of the confinement potential range in the lateral directions x, y and vertical direction z, respectively. For $p = 2$ we obtain the Gaussian potential and for $p > 10$ the shape of the confinement potential resembles the rectangular potential well.

Electrons confined in the quantum dot form localized bound states with discrete energy levels. These states exhibit a qualitative similarity to the quantum states of natural atoms. Therefore, the quantum dots are sometimes called artificial atoms. The two quantum dots, which are coupled by the tunnel barrier, form an artificial molecule. From the point of view of a possible application to quantum computation, the single-electron transport via the quantum dot is of crucial importance. The main single-electron transport channel is the sequential tunneling, in which the single electrons tunnel through the dot in subsequent time intervals provided the transport conditions are fulfilled [27]. The single-electron transport measurements appeared to be the successful spectroscopic method, which allowed to discover the wonderful properties of quantum dots: the filling of the shells of artificial atoms [23] and the quantum Coulomb blockade [28]. The quantitative theoretical description of these effects was given in paper [21]. The vertical gated quantum-dot nanodevice [23] is a prototype of a single-electron transistor, which can be switched on and off by the flow of the single electron. In future, the application of single-electron transistors will lead to a much higher performance of electronic devices at greatly reduced power consumption.

Recently, the possibility of implementation of quantum dots to quantum computation is intensively studied [17, 18]. The qubits can be realized as either the charge states or spin states of the quantum dots. The electrostatic quantum dots seem to be especially well suited to perform the quantum computations, since their electronic properties can be modelled by the proper choice of the nanostructure parameters and tuned by changing the external voltages applied to the electrodes. This enables both to obtain the designed properties of the quantum states (quantum engineering) and perform the controlled logic operations on these states. Moreover, the modern nanotechnology of fabrication of quantum dots is an extension toward a smaller feature size of the well known semiconductor MOSFET technology. Therefore, its introduction into the production is more easy than those of the other quantum-computing technologies, based on ion traps and QED cavities, which are obtained only in advanced laboratories.

4. Model of implementation of qubits and logic gates with quantum dots

In quantum dots, the qubits can be formed from the quantum states of electrons or excitons. The exciton is the bound electron-hole system, which is created in the absorption of the photon with energy comparable with the forbidden energy gap of the semiconductor. After the lifetime of the order of microseconds, the exciton recombinates with the emission of the photon. The excitonic qubit can be realized in a simple manner: we ascribe state $|1\rangle$ to the existing exciton in the ground state and state $|0\rangle$ to the system after recombination, i.e., an empty conduction band, a fully filled valence band, and a photon. This concept has a disadvantage of the short decay time (lifetime) of the exciton. However, it has the important advantage of the easy performance of the read/write operations with the help of visible light photons in emission/absorption processes. A possible implementation of biexcitons (bound two-exciton systems) as the qubits has also been studied [29].

Nowadays, it seems that the most promising for quantum computation is the application of the spin states of quantum-dot confined electrons. The electron spin states possess the following advantages: very long relaxation time [17] in the absence of external fields, fairly long decoherence time (the experiments [30] suggest $t_{decoh} \simeq 1\mu$s), and the possibility of easy manipulation of the spin by the external magnetic field. The research toward the implementation of the electron spin as a new information carrier is the subject of a new electronics based on spin, called spintronics [32, 33]. Recently, the spin transistor has been designed [34, 35]. We can use the electron spin states to construct qubits and logic operations on them in two ways, either directly with the application of the spin magnetic dipol coupling with the magnetic field (cf. Subsection 2.2) or indirectly with the application of symmetry properties of the many-electron wave function. The indirect application of spin to represent the qubit is based on the change of the sign of the wave function during the exchange of the space-spin coordinates of two electrons. This fundamental quantum-statistical property leads to the well defined symmetry of the spin states in a subspace of states with the definite spin. For example, in the two-electron system, the spin singlet state (antisymmetric against the spin exchange) possesses a different (usually lower) energy than the spin triplet states (symmetric with respect to the spin exchange). The resulting singlet-triplet energy level splitting can be used to distinguish the spin qubits and to perform logic operations on them, e.g., with the use of photon emission and absorption.

In the present section, we discuss a modification of the proposition [19] of an implementation of spin states of electrons confined in coupled quantum dots in a quantum computation. In this case, the computational basis is formed from the eigenstates of the z component of the electron spin (cf. Subsection 2.2). Due to the universal character of the controlled NOT gate we will discuss its realization with the use of the two coupled quantum dots. The operation of the CNOT gate is defined by Eq. (14).

Let us consider the two-electron system in the two coupled quantum dots labelled by index $j = 1, 2$. We assume that the single electron is confined in each dot. If the quantum dots are fabricated from the different materials, the effective Lande factors are different, i.e., $g_1^\star \neq g_2^\star$. According to Eq. (9), this leads to the different Zeeman energy of the electron in each dot. The similar effect can be achieved by applying the inhomogeneous magnetic field, which takes on the different value in each of the dots. The material composition and the thickness of the barrier layer, which separates the quantum dots, is chosen so that a small coupling between the dots is ensured. The Hamiltonian of the system considered has the form

$$H = H_1 + H_2 + H_{int} \; , \tag{18}$$

where H_j $(j = 1, 2)$ is the one-particle Hamiltonian of the electron with spin $s_{z,j}$ in the external magnetic field, i.e.,

$$H_j = \omega_j s_{z,j} \; , \tag{19}$$

where $\omega_j = g_j^\star \mu_B B_j/\hbar$ [cf. Eq. (8)] and the magnetic field in the jth quantum dot is given by $\mathbf{B}_j = (0, 0, B_j)$. The interaction Hamiltonian has the form

$$H_{int} = (4/\hbar)\Omega s_{z,1} s_{z,2} \; , \tag{20}$$

where parameter Ω characterizes the coupling between the electron spins in different quantum dots. The spin operators $s_{z,1}$ fulfill the eigenequations

$$s_{z,1}|0, l\rangle = +\frac{\hbar}{2}|0, l\rangle \; , \;\; s_{z,1}|1, l\rangle = -\frac{\hbar}{2}|1, l\rangle \; , \tag{21}$$

where $|k, l\rangle$ are the two-spin states ($|k, l\rangle = |k\rangle \otimes |l\rangle$) and $k, l = 0, 1$. The similar two equations are satisfied for the operators $s_{z,2}$. According to the above assumptions $\omega_1 \neq \omega_2$.

In the absence of coupling, i.e., for $\Omega = 0$, Hamiltonian (18) possesses the following eigenvalues: $\epsilon_1 = -(\hbar/2)(\omega_1 + \omega_2)$ in state $|1, 1\rangle$, $\epsilon_2 = -(\hbar/2)(\omega_1 - \omega_2)$ in state $|1, 0\rangle$, $\epsilon_3 = (\hbar/2)(\omega_1 - \omega_2)$ in state $|0, 1\rangle$, and $\epsilon_4 = (\hbar/2)(\omega_1 + \omega_2)$ in state $|0, 0\rangle$. The coupling can be switched off by applying the barrier layer with sufficiently large thickness. Each spin state can be selectively addressed in processes of absorption or emission of photons with the proper frequency. For example, in the absorption process, the photon with frequency ω_1 operates exclusively on the first qubit changing it from state $|1, l\rangle$ (spin-down state) into state $|0, l\rangle$ (spin-up state). The reverse process, i.e., $|0, l\rangle \to |1, l\rangle$, is also possible in the stimulated emission. These processes occur for each $l = 0, 1$, i.e., for an arbitrary state of the second qubit. Similarly, the photon with frequency ω_2 will change the state of the second qubit only.

The case of the coupling switched on ($\Omega > 0$) can be described using the solutions to the eigenproblem of Hamiltonian (18). It appears that the two-spin basis states

$$\{|1,1\rangle, |1,0\rangle, |0,1\rangle, |0,0\rangle\} \tag{22}$$

are the eigenstates of Hamiltonian (18) to the eigenvalues $E_1 = \epsilon_1 + \hbar\Omega$, $E_2 = \epsilon_2 - \hbar\Omega$, $E_3 = \epsilon_3 - \hbar\Omega$, $E_4 = \epsilon_4 + \hbar\Omega$, respectively. This means that after switching on the interdot coupling the energy levels are changed by $\pm\hbar\Omega$, while the eigenstates remain unchanged. This enables us to realize the selected transitions between the basis states (22). The resonant radiation with accurately chosen frequency causes the switching on and off between basis states (22) associated with the change of one qubit depending on the state of the second qubit. For example, the photon with frequency $\omega_2 - 2\Omega$ induces the switching between states $|1, 0\rangle$ and $|1, 1\rangle$ only, leaving states $|0, 0\rangle$ and $|0, 1\rangle$ unchanged. Therefore, in the stimulated emission/absorption processes induced by this photon, we can implement the CNOT operation.

This proposition of the implementation of the CNOT logic gate is based on a simplified model of interaction between the spins. On the other hand, interaction Hamiltonian (20) possesses the form of the Heisenberg Hamiltonian, which is an universal Hamiltonian of interaction between particles with spin. The Heisenberg Hamiltonian can be used to a description of the interaction between different physical objects with non-zero spin, e.g., electrons, ions, and atomic nuclei. The model of the CNOT gate, discussed above, can be realized in the coupled quantum dots as well as in the NMR system.

We note that the other methods are also studied for the implementation of spin qubits in quantum dots [32]. They include: (i) a measurement of spin via the measurement of charge, (ii) a measurement of a spontaneous magnetisation of the quantum dot, (iii) electron spin resonance, and (iv) a measurement of singlet-triplet splitting with the help of a Faraday rotation. Method (i) employs the spin filter, which generates a current of spin polarized electrons through a semiconductor. Recently, a very interesting realization of qubits on quantum dots has been experimentally studied [36] with the direct use of the electron charge. The authors [36] succeeded in forming the qubits in the coupled quantum dots as the states with the presence or absence of the single electron in the first or the second dot. This achievement is very promising since it enables us to construct the qubits by charging the quantum dot and detect them by measuring the electric charge.

5. Conclusions and summary

The present stage of research toward a construction of a quantum computer can be characterized as follows: the theory of quantum computation, based on the solid foundations of quantum mechanics, is very advanced, while the practical realization of the quantum computation, although being a subject of intensive experimental studies in physical laboratories, is in an emerging stage. The quantum computing theory comprises the elements of quantum physics, mathematics, and information theory. This theory shows that it is possible to apply directly the quantum effects to a superdense information storage and massively parallel computing with a very high speed. Nowadays, we face the problem of the physical realization of the quantum computer. Besides the quantum-dot quantum computer, treated in the present article, the different competitive technologies are intensively studied. The most promising of them are ion traps, QED cavities, NMR systems, and superconducting systems.

The spin of quantum particles (electron, atomic nucleus, photon) is especially suitable to construct of and operate with the qubits. In the field of semiconductor physics and electronics, the application of electron spin as an information carrier has led to an emergence of spin electronics (spintronics) [32, 33]. On the other hand, the miniaturization of semiconductor devices resulted in a development of nanoelectronics. In the nanoelectronic devices, the charge of single electrons is used to carry on and transform the information. The quantum dots are very important nanostructures from the point of view of both the spintronics and nanoelectronics. They possess the important advantage, namely, the possibility of tuning and controlling their electronic properties by changing the external electromagnetic fields. This allows us to obtain the required characteristics and modify them in a very short time. It seems that – due to their flexibility – the quantum dots are promising candidates as elements of future quantum computers. The other important characteristics of quantum-dot nanodevices is their compatibility with the existing hardware. It is important for the emerging quantum technology that the construction of the spin- and nanoelectronic devices is based on the progress in the existing electronics and the present semiconductor industry.

In summary, nowadays we have at disposal the advanced quantum computation theory. However, the physical implementation of quantum computation is in the stage of laboratory studies. The electron spin states in the coupled quantum dots, discussed in the present paper, are good candidates for the qubits. Moreover, the quantum logic gates can be implemented with these states.

6. Acknowledgements

This paper has been partly supported by the Polish Ministry for Scientific Research and Information Technology in the framework of the solicited grant PBZ-MIN-008-/P03/2003. One of us (B.S.) is supported by the Foundation for Polish Science (FNP).

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Received February 24, 2004